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# Dynamic symbol Allocation for Spectral and Energy Efficient Millimeter Wave Multi-Antenna Systems

Atiqa Kayani, Ijaz Haider Naqvi and Muhammad Zeeshan Shakir

In this letter, we investigate the relationship between number of parallel transmit symbols and the spectral efficiency of a millimeter wave (downlink) massive MIMO system with multiple antenna array sizes and a hybrid beamforming strategy. The proposed approach determines the optimal number of parallel data symbols that maximizes both the spectral and energy efficiency. A heuristic algorithm is proposed that adapts the number of symbols at the transmitter based on the instantaneous channel state information. The proposed algorithm outperforms non-adaptive strategies for multiple array sizes and achieves a significant improvement in spectral and energy efficiency.

**Introduction:** High throughput requirements and scarcity of available spectrum calls for exploration of millimeter wave (mm-Wave) frequency band [1]. However, mm-Wave signals experience an order of magnitude higher pathloss compared to the majority of current microwave wireless systems. This higher pathloss can be compensated by making use of the multiple antenna arrays both at the transmitter and the receiver. The use of large antenna arrays allows precoding multiple data symbols simultaneously thereby improving both spectral and energy efficiency [2]. For this purpose, different types of precoding schemes have been proposed such as digital, analog and hybrid. Hybrid architecture, which is a combination of analog and digital precoders, provides near optimal performance with limited number of RF chains [3]. It is a cascade of analog and digital precoders with the ability to transmit multiple data symbols.

The prior art on the design of hybrid precoders focuses only on fixed number of data symbols. For mm-Wave multi-antenna systems, the antenna elements are tightly packed which corresponds to a higher spatial correlation amongst different antenna elements [4]. This relatively large correlation limits the number of independent fading channels. As a result, the spectral efficiency is not a monotonically increasing function of transmitted data symbols. To the best of authors' knowledge, there has been no concrete work on the optimization of data symbols and dynamic symbol allocation in the literature.

In this letter, the number of transmitted data symbols are dynamically determined based on the channel conditions for multi-antenna mm-Wave communication system. The optimal number of data symbols that maximizes both spectral and energy efficiency is heuristically determined. The simulation results validate the superiority of the proposed approach in comparison to the precoder designs with fixed number of data symbols.

**System Model:** We consider a single user mm-Wave massive MIMO system with hybrid architecture. The system comprises of  $N_t$  transmit antennas,  $N_r$  receiving antennas and communicates  $N_s$  data symbols as shown in Fig. 1. The number of data symbols are varied based on the channel conditions and are first passed through the baseband (BB)/digital precoding stage. The BB precoded data is then passed through an RF (analog) stage where it is passed through  $N_t^{RF}$  RF chains followed by an analog (RF) precoder before being transmitted by  $N_t$  transmitting antennas [3]. Note that we can at most transmit  $N_s \leq N_t^{RF} \leq N_t$  data symbols through our system. Similarly, the receiver must satisfy the constraint  $N_s \leq N_r^{RF} \leq N_r$ . The transmitted symbol vector  $\mathbf{s}$  is generated in such a way that  $\mathbb{E}[\mathbf{s}\mathbf{s}^*] = \frac{1}{N_s} \mathbf{I}_{N_s}$ , where  $\mathbf{I}_{N_s}$  is an identity matrix of size  $N_s$  and  $(\cdot)^*$  denotes the conjugate transpose. In general, the received signal can be represented as

$$\mathbf{y} = \sqrt{\rho_r} \mathbf{H} \mathbf{x} + \mathbf{n}, \quad (1)$$

where  $\mathbf{y}$  is  $N_r \times 1$  received signal vector,  $\rho_r$  is the averaged received signal power,  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$  represents the channel matrix,  $\mathbf{x} = \mathbf{P}_{\text{RF}} \mathbf{P}_{\text{BB}} \mathbf{s}$  is the transmitted signal of size  $N_t \times 1$  and  $\mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2)$  is a circularly symmetric complex Gaussian noise vector. Also note that  $\mathbf{P}_{\text{RF}} \in \mathbb{C}^{N_t \times N_t^{RF}}$  and  $\mathbf{P}_{\text{BB}} \in \mathbb{C}^{N_t^{RF} \times N_s}$  are the RF and baseband precoder matrices respectively. As RF precoders are implemented using phase shifters, they have to obey constant modulus constraint, i.e.,  $(\mathbf{P}_{\text{RF}} \mathbf{P}_{\text{RF}}^*)_{ii} = 1/N_t$ . The power constraint at the transmitter is satisfied by ensuring  $\|\mathbf{P}_{\text{RF}} \mathbf{P}_{\text{BB}}\|_F^2 \leq N_s$ , where  $\|\cdot\|_F$

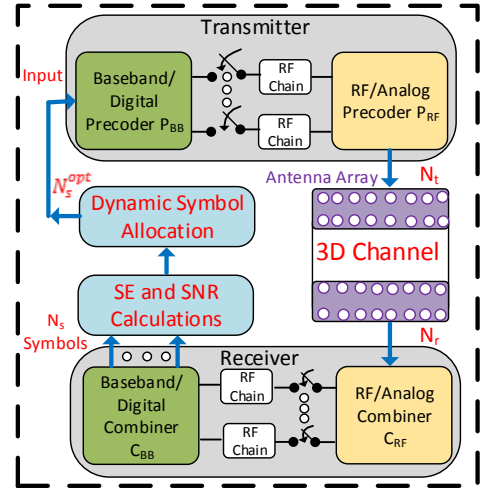


Fig. 1 A block diagram of Millimeter wave (mm-Wave) massive MIMO system with hybrid precoding architecture

represents the frobenius norm. It is also assumed that the instantaneous channel gains are perfectly known both at the transmitter and the receiver.

At the receiver  $\hat{\mathbf{y}} = \mathbf{C}_{\text{BB}}^* \mathbf{C}_{\text{RF}}^* \mathbf{y}$ , where received signal  $\mathbf{y}$  is processed by analog combiner  $\mathbf{C}_{\text{RF}} \in \mathbb{C}^{N_r \times N_r^{RF}}$  at the RF stage followed by a digital combiner  $\mathbf{C}_{\text{BB}} \in \mathbb{C}^{N_r^{RF} \times N_s}$  in the baseband. We consider a narrow-band clustered channel model for mm-Wave massive MIMO system, given as [5]

$$\mathbf{H}_{3D} = \zeta \sum_{i,l}^{N_L, N_c} \beta_{il} \Lambda_r(\phi_{il}^r, \theta_{il}^r) \Lambda_t(\phi_{il}^t, \theta_{il}^t) \mathbf{a}_r(\phi_{il}^r, \theta_{il}^r) \mathbf{a}_t(\phi_{il}^t, \theta_{il}^t)^*, \quad (2)$$

where  $\zeta = \sqrt{\frac{N_t N_r}{N_L N_c}}$  is the normalization factor that makes sure that  $\mathbb{E}[\|\mathbf{H}\|_F^2] = N_t N_r$ ,  $N_c$  is the number of scattering clusters,  $N_L$  is the number of multipath components in each of the clusters,  $\beta_{il} \sim \mathcal{CN}(0, 1)$  is the complex gain of the  $l^{\text{th}}$  path and  $i^{\text{th}}$  scattering cluster,  $\mathbf{a}_r(\phi_{il}^r, \theta_{il}^r)$  and  $\mathbf{a}_t(\phi_{il}^t, \theta_{il}^t)$  are the normalized receive and transmit array response vectors respectively,  $\phi_{il}^r$  and  $\theta_{il}^r$  are the azimuth and elevation angles of arrival (AoA) at the receiver and finally  $\phi_{il}^t$  and  $\theta_{il}^t$  are the azimuth and elevation angles of departure (AoD) at the transmitter. All these angles are uniformly distributed between  $[-\pi/2, \pi/2]$ . We have assumed an omni-directional radiation pattern i.e.,  $\Lambda_r(\phi_{il}^r, \theta_{il}^r) = \Lambda_t(\phi_{il}^t, \theta_{il}^t) = 1$ . For a uniform planar array (UPA), the array response vector is given by

$$\mathbf{a}_{\text{UPA}}(\phi, \theta) = \frac{1}{\sqrt{N}} [1, \dots, e^{jkd(a \sin(\phi) \sin(\theta) + b \cos(\theta))}, \dots, e^{jkd((A-1) \sin(\phi) \sin(\theta) + (B-1) \cos(\theta))}]^T, \quad (3)$$

where  $N = AB$  is the array size and indices  $0 \leq a < A$  and  $0 \leq b < B$  correspond to the antenna elements along  $y$  and  $z$  axis respectively.

**Precoder Design:** Optimal digital precoders require as many RF chains as the number of antenna elements. The cost and power consumption of RF chains make fully digital precoder infeasible for practical mm-Wave massive MIMO systems. Therefore, hybrid precoding structures with reduced number of RF chains and near optimal performance have recently been proposed as an alternative [3]. Hybrid architecture splits the processing into two blocks (RF and BB) and significantly reduces the required number of RF chains [7]. The hybrid precoders are designed by solving matrix factorization problem by using standard algorithms like orthogonal matching pursuit (OMP) etc.

**Problem Formulation and Proposed Algorithm:** We investigate the problem of dynamic symbol allocation for hybrid architecture to maximize the spectral efficiency. The spectral efficiency  $\eta$  for mm-Wave massive MIMO system is given as [6]

$$\eta = \log_2 \left( \left| \left( \mathbf{I}_{N_s} + \frac{\rho_r}{N_s} \mathbf{R}_n^{-1} \mathbf{C}_{\text{BB}}^* \mathbf{C}_{\text{RF}}^* \mathbf{H} \mathbf{P}_{\text{RF}} \mathbf{P}_{\text{BB}} \times \mathbf{P}_{\text{BB}}^* \mathbf{P}_{\text{RF}}^* \mathbf{H}^* \mathbf{C}_{\text{RF}} \mathbf{C}_{\text{BB}} \right) \right| \right), \quad (4)$$

where  $\mathbf{R}_n \triangleq \sigma_n^2 (\mathbf{C}_{\text{BB}}^* \mathbf{C}_{\text{RF}}^* \mathbf{C}_{\text{RF}} \mathbf{C}_{\text{BB}})$  is the noise covariance matrix obtained after combining operation. The received SINR is a function of number of transmitted data symbols. If  $1 \leq N_k \leq N_s$  data symbols are transmitted, signal-to-interference-plus-noise ratio (SINR) is expressed

as

$$\Gamma_{N_k} = \frac{\rho_r}{N_s} \frac{|C(:,k)^* \mathbf{H} \mathbf{P}(:,k)|^2}{\sum_{i=1, i \neq k}^{N_s} |C(:,i)^* \mathbf{H} \mathbf{P}(:,i)|^2 + \sigma_n^2 \|C(:,k)\|^2}, \quad (5)$$

where  $\mathbf{P} \triangleq \mathbf{P}_{\text{RF}} \mathbf{P}_{\text{BB}}$  and  $\mathbf{C} \triangleq \mathbf{C}_{\text{RF}} \mathbf{C}_{\text{BB}}$ . The spectral efficiency with  $N_k$  data symbols is written as

$$\eta_{N_k} = \log_2(|\mathbf{I}_{N_k} + \Gamma_{N_k}|). \quad (6)$$

The optimal spectral efficiency is determined by solving the following optimization problem

$$\begin{aligned} & \underset{N_s}{\text{maximize}} \quad \eta \\ & \text{subject to} \quad N_s \leq N_{\text{RF}}^t \leq N_t, \\ & \quad N_s \leq N_{\text{RF}}^r \leq N_r, \\ & \quad \eta_{N_k} \geq \eta_{th}, \end{aligned} \quad (7)$$

where  $\eta_{th}$  is the required quality of service (QoS) constraint that specifies the information rate (bits/s/Hz) if  $N_k$  data symbols are transmitted. The above problem is NP-hard and there exists no standard method to solve such an optimization problems. Thus, a heuristic algorithm is used to determine the number of transmitter data symbols to maximize the instantaneous spectral efficiency for each channel realization.

For each channel realization, the algorithm computes the spectral efficiency for all  $N_k$  data symbols and declares  $N_s^{\text{opt}} = \max(N_k)$  for  $k \in \{1, \dots, N_s\}$ . The  $N_k$  that maximizes the spectral efficiency is declared as  $N_s^{\text{opt}}$ . This information is fed-back to the transmitter that dynamically adjusts the number of parallel transmissions (see Fig. 1). This entire procedure is summarized in Algorithm 1. Moreover, by optimizing the number of data symbols, reduced number of RF chains are used compared to non adaptive strategy. Hence, the proposed approach is much more energy efficient (typically measured in bits/Hz/Joules).

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#### Algorithm 1 Optimal symbol allocation algorithm

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**Inputs:**  $C, P$

**Output:**  $N_s^{\text{opt}}$

**Initialization:**  $N_s = 1, 2, \dots, N_{\text{RF}}^t$ , Range of values for  $\sigma_n^2$

**For**  $i=1:\text{length}(\sigma_n^2)$  **do**

**For**  $k=1:N_s$ , **do**

Compute SINR for all  $k$  parallel data symbols

Compute SE,  $\eta_{N_k}$ , for all  $k$  parallel data symbols

**End For**

return  $N_k = N_s^{\text{opt}}$  for which  $\eta_{N_k} = \max(\eta_{N_k})$

**if**  $\eta_{N_s^{\text{opt}}} < \eta_{th}$ , then Do Not transmit; wait for next channel realization

**End if**

**End For**

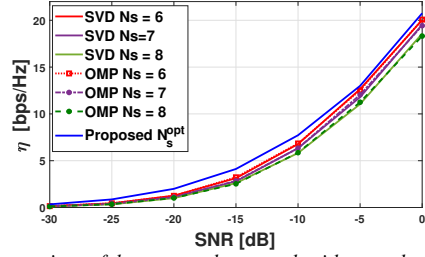
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Run Monte Carlo simulations for  $N_s^{\text{opt}}$  data symbols

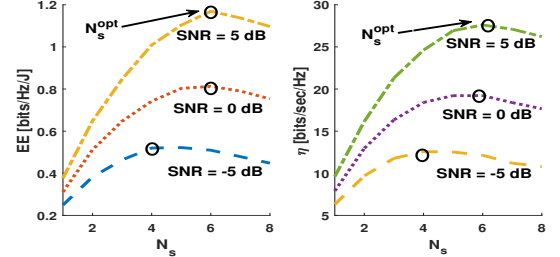
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**Results and Discussions:** The simulations are carried out with  $N_t = \{8, 16, 128\}$  transmit antennas and  $N_r = \{8, 16\}$  receive antennas which are half wavelength apart. A single cluster mm-Wave (28 GHz) channel [5] with 8 sub-paths is used for simulations. Similar results are attained for all these configurations but only the results for  $128 \times 16$  system are presented in the interest of space. Fig. 2 shows the spectral efficiency of a  $128 \times 16$  antenna system with UPA, used at both the transmitter and receiver, with the proposed approach and non adaptive fully digital (SVD) and hybrid OMP architectures. For the hybrid design,  $N_{\text{RF}}^t = N_{\text{RF}}^r = 8$  i.e., the system can transmit up to 8 data symbols in parallel. The results are truly remarkable. The fact that the proposed approach outperforms fully digital SVD architecture (considered as optimal) is an outstanding achievement of the proposed strategy that uses significantly lower energy and fewer number of RF chains. Fig. 3 shows the comparison of spectral and energy efficiency as a function of data symbols for three different values of SNR for  $128 \times 16$  system. The fact that the optimal number of data symbols change with the received SNR validates the idea of dynamic symbol allocation. Note that, the heuristic algorithm only looks for an optimal spectral efficiency, however, the non conflicting nature of SE and EE leads to the same  $N_s^{\text{opt}}$  for both objectives.

In order to quantify the gains of dynamic symbol allocation, we dynamically vary the number of data symbols and transmit  $N_s^{\text{opt}}$  parallel data symbols across different channel conditions for  $8 \times 8$  and  $128 \times 16$  systems. Table 1 compares the spectral efficiency (SE) and energy efficiency (EE) achieved with fixed and dynamic ( $N_s^{\text{opt}}$ ) number of



**Fig. 2** SE comparison of the proposed approach with non adaptive fully digital and hybrid OMP schemes for  $128 \times 16$  system with  $N_{\text{RF}} = 8$ .



**Fig. 3** Achieved EE and SE vs.  $N_s$  for  $128 \times 16$  system for three different values of SNR.

**Table 1:** SE and EE for multiple fixed symbol and the proposed dynamic symbols allocation strategy for  $8 \times 8$  and  $128 \times 16$  system.

$(N_s)$	SE ( $8 \times 8$ )	EE ( $8 \times 8$ )	SE ( $128 \times 16$ )	EE ( $128 \times 16$ )
1.	0.4381	0.1352	0.8622	0.0347
2.	0.5599	0.1728	1.4119	0.0568
3.	0.5521	0.1704	1.7858	0.0719
4.	0.5085	0.1570	2.0521	0.0826
6.	—	—	2.1903	0.0882
Opt.	1.1508	0.4567	5.2522	0.2114

transmitted data symbols averaged over SNR varying from -40 dB to 0 dB. The proposed transmission strategy achieves the highest SE and EE for all antenna arrangements. For spectral efficiency, it achieves staggering 105% and 140% better performance than the next best of the non adaptive strategy for  $8 \times 8$  and  $128 \times 16$  systems respectively. Similarly, the comparison of energy efficiency reveals that the proposed scheme gives 164% and 140% improved performance for  $8 \times 8$  and  $128 \times 16$  systems respectively.

**Conclusion:** In this letter, we propose a novel symbol allocation strategy for mm-Wave, multi-antenna system that maximizes both spectral and energy efficiency. The number of transmitted symbols are dynamically varied based on the channel conditions to achieve optimal performance. The proposed transmission strategy outperforms the best amongst fixed symbol transmission with gains up to 140% and 164% for spectral and energy efficiency respectively.

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